$$F_i = L_i \circ H_i \Rightarrow F_i'(x) = L_i H_i'(x) \Rightarrow F_i'(\tilde{x}) = L_i \underbrace{H_i'(\tilde{x})}_{= H_i'(x) R_{i\tilde{x}}^{x}} = F_i'(x) R_{i\tilde{x}}^{x}$$

$$H_i'(\tilde{x}) = H_i'(\tilde{x}) \underbrace{H_i'(x)^{-1} H_i'(\tilde{x})}_{=: R_{i\tilde{x}}^{x}} = H_i'(x) R_{i\tilde{x}}^{x}$$

$$\| R_{i\tilde{x}}^{x} - I \| = \| H_i'(x)^{-1} (H_i'(\tilde{x}) - H_i'(x)) \| \leq C_i L_i \| \tilde{x} - x \| \quad \forall i$$

$$C_R := \max \{ C_i L_i : i \in \{0, \dots, N-1\} \}$$

Ex 1 $\quad F_i : L^2(\Omega) \to H^{\frac{1}{2}}(\partial\Omega) \quad q \mapsto \frac{\partial u}{\partial \nu}\Big|_{\partial\Omega}$ s.t. $\begin{cases} -\Delta u + q u = 0 & \text{in } \Omega \\ u = f_i & \text{on } \partial\Omega \end{cases}$

$w := F_i(\tilde{q}) - F_i(q)$:

$$\begin{aligned} -\Delta \tilde{u} + \tilde{q} \tilde{u} &= 0 \\ -\Delta u + q u &= 0 \\ \hline -\Delta(\tilde{u} - u) + \tilde{q}(\tilde{u} - u) &= -(\tilde{q} - q) u \end{aligned}$$

$\Rightarrow F'(q)h =: v$ solves $\begin{cases} -\Delta v + q v = -h u & \text{in } \Omega \\ v = 0 & \text{on } \partial\Omega \end{cases}$

$A_q : H^2(\Omega) \cap H_0^1(\Omega) \to L^2(\Omega)$

$\qquad v \mapsto -\Delta v + q v$

$$\Rightarrow F'(q) h = -A_q^{-1}[h \cdot u]$$

$$\begin{aligned} F'(\tilde{q}) h &= -A_{\tilde{q}}^{-1}[h \cdot \tilde{u}] \\ &= -A_q^{-1} A_q A_{\tilde{q}}^{-1} [h \cdot \tilde{u}] \\ &= -A_q^{-1} [R_{\tilde{q}}^{q} h \cdot u] \end{aligned}$$

$$R_{\tilde{q}}^{q} h = \frac{1}{u} A_q A_{\tilde{q}}^{-1} [h \cdot \tilde{u}]$$

$$\begin{aligned} (R_{\tilde{q}}^{q} - I) h &= \frac{1}{u} A_q A_{\tilde{q}}^{-1} [h \cdot \tilde{u}] - \frac{1}{u} A_q A_q^{-1} [h \cdot u] \\ &= \frac{1}{u} A_q \{ \underbrace{(A_{\tilde{q}}^{-1} - A_q^{-1})}_{= A_q^{-1}(A_q - A_{\tilde{q}}) A_{\tilde{q}}^{-1}} [h \cdot \tilde{u}] + A_q^{-1} [h \cdot (\tilde{u} - u)] \} \\ &= \frac{1}{u} \{ (q - \tilde{q}) A_{\tilde{q}}^{-1} [h \cdot \tilde{u}] + h \cdot \underbrace{(\tilde{u} - u)}_{= w = -A_{\tilde{q}}^{-1}[(\tilde{q} - q) u]} \} \\ &= -\frac{1}{u} \{ (\tilde{q} - q) A_{\tilde{q}}^{-1} [h \cdot \tilde{u}] + h A_{\tilde{q}}^{-1} [(\tilde{q} - q) \cdot u] \} \end{aligned}$$

$\rightsquigarrow$ need u bounded away from zero

$$u_{tt} = c^2 \Delta u \underset{\mathcal{F}}{\Longleftrightarrow} \Delta \hat{u} + \left(\frac{\omega}{c}\right)^2 u = 0 \quad \text{in } \Omega$$

$$\underset{\substack{c_0 := \frac{1}{|\Omega|}\int_\Omega c(x)\,dx \\ f := 1-\left(\frac{c_0}{c}\right)^2,\, k = \frac{\omega}{c_0}}}{\Longleftrightarrow} \quad \Delta \hat{u} + k^2(1-f)\hat{u} = 0 \quad \text{in } \Omega$$

$$\hat{u}(x) = \underbrace{e^{ikx\cdot\theta_i}}_{\hat{u}_{in}(x)} + \hat{v}(x) : \quad \Delta \hat{u} + k^2(1-f)\hat{u} = 0$$

$$\Delta \hat{u}_i + k^2 \hat{u}_{in} = 0 \Leftrightarrow \Delta \hat{u}_{in} + k^2(1-f)\hat{u}_{in} = -k^2 f \hat{u}_{in}$$

$$\Delta \hat{v} + k^2(1-f)\hat{v} = -k^2 f \hat{u}_{in}$$