Exercise Sheet 2

Asymptotic regularization

Asymptotic regularization (also called *Showalter's method*) consists of solving the initial value problem

$$\begin{aligned} u_\delta'(t) + T^*Tu_\delta(t) &= T^*y^\delta, \qquad t > 0 \\ u_\delta(0) &= 0 \end{aligned} \tag{1}$$

and approximating $x^\dagger = T^\dagger y$ by $x_\alpha^\delta = R_\alpha y^\delta = u_\delta(1/\alpha)$, i.e., the stopping time plays the role of a regularization parameter.

1. Prove that the unique solution $u_\delta \in C^1([0,\infty), X)$ of the initial value problem (1) is given by
$$u_\delta(t) = (T^*T)^{-1}(I - \exp(-tT^*T))T^*y^\delta.$$

2. Determine the functions q_α und r_α for Showalter's method and verify conditions (11), (12), (13), and (18) (with $\mu_0 = \infty$)

3. Prove that: Solving the initial value problem (1) by an explicit Euler method with step size one yields Landweber iteration.

4. Prove that: Solving the initial value problem (1) by an implicit Euler method with step size $1/\alpha$ yields iterated Tikhonov regularization.